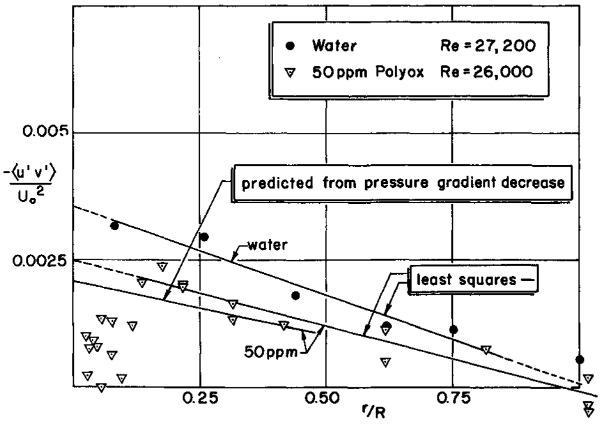

Fig. 5 Reynolds stress across the pipe.

extrapolated line based on the observed pressure drop at the wall, relative to the observed pressure drop for water.

Of considerable interest is the marked reduction of $\langle u'v' \rangle$ for the polymer solution in the region $0 < r/R < 0.125$. This seems to indicate that u' and v' are very greatly out of phase to a distance of approximately twice that of the $\langle u'^2 \rangle^{1/2}$ peak. It is hoped that future explanations of these and subsequent measurements will help provide a basis for increased understanding of the mechanism of drag reduction by polymer additives.

References

[1] Toms, B. A., "Some Observations on the Flow of Linear Polymer Solutions Through Straight Tubes at Large Reynolds Numbers," *Proceedings of First International Rheological Congress,* Holland, 1948, pp. 135–141.

[2] Virk, P. S. et al., "The Toms Phenomenon: Turbulent Pipe Flow of Dilute Polymer Solutions," *Journal of Fluid Mechanics,* Vol. 30, Pt. 2, Nov. 1967, pp. 305–328.

[3] Rudd, M. J., "Measurements Made on a Drag Reducing Solution With a Laser Velocimeter," *Nature,* Vol. 224, Nov. 1969, pp. 587–588.

[4] Logan, S. E., "A Laser Velocimeter for Reynolds Stress and Other Turbulence Measurements," *AIAA Journal,* to be published.

Rational Reduction of Large-Scale Eigenvalue Problems

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Introduction

EIGENVALUE analysis of the large dynamic matrices associated with complex structures can be computationally costly and unwieldy. Usually some form of "condensation" is carried out to reduce the matrix sizes involved[1,2] and the cost of determining the modest number of eigenvalues generally required. The most appropriate freedoms to be condensed from the total can be partly determined by judgement (e.g., rotational freedoms in a beam element), but in a "compact," complex structure it is difficult to decide on a completely rational and adequate division of retained and condensed freedoms.

There is, therefore, a need for a rational, scientific approach to the determination of the optimum set of freedoms to retain, which merely leaves the analyst to specify the number of freedoms to be retained. Such a procedure must justify itself economically in the total problem of arriving at the required information.

In Ref. 3, Lanczos' iteration procedure[4] was used to reduce a large system to a reduced rank tridiagonal form. Although improved numerical stability was thereby achieved, the selection of transformation vectors satisfying orthogonality conditions required considerable computational effort.

In this Note, transformation vectors are selected from coordinates which determine the largest (approximate) "swept volume" (area times deformation) generated by the local inertia force per unit acceleration. These are termed "maximum volume" modes, and, with attention to independency, form the basis for the rational choice of freedoms to be retained or condensed.

The deformation modes and their selection involve small computational effort typical only of "static" solutions. Basic principles for supported and free-free structures are outlined and a few simple examples illustrated.

Approach

Any eigenfunction can be approximated as a linear combination of "static" deformation modes. To extract the most significant freedoms for the determination of a subset of lowest eigenvalues weighted consideration of both inertial and flexibility characteristics is required.

Let

$$Kq = -M\ddot{q} \tag{1}$$

where

$K = n \times n$ symmetric stiffness matrix } of the complete
$M = n \times n$ symmetric inertia matrix } structural model

and let K be resolved into upper and lower triangular matrices L, U, i.e., such that

$$K = LU \tag{2}$$

and let M_p = a rectangular $(n \times p)$ mass matrix with respect to potentially "significant" inertia loads, i.e., excluding those obviously ripe for condensation.

M_p may range from the complete inertia matrix down to the consideration of nonstructural inertias (including contained fluids) only.

Assuming in turn $\ddot{q}_p = 1$, a set of deformation vectors can be generated by the usual static analysis routine (again using L, U) which may be written

$$[D]_p = [LU]^{-1}[M]_p \tag{3}$$

Maximum volume deformation modes are now determined by ranking the norms of the vectors of $[D]_p$, i.e.,

$$Q_i = \left(\sum_j D_{ij}^2\right)^{1/2} \quad \text{for} \quad i,j = 1 \ldots p \tag{4}$$

Let "r" ($\ll n, p$) be the rough number of deformation modes required to determine "s" eigenvalues. Then a good choice of columns appears to be the "r" largest of the Q_i's.

The fundamental success of the method will be judged by the smallness of the ratio (r/s). In an ideal condensation scheme, this should approach unity. Present methods[1,2] seem to require $r \div s > 3$.

Collect r columns of $[D]_p$ *as a subset T,* and now define

$$\underset{(n\times 1)}{q} = \underset{(n\times r)}{T} \cdot \underset{(r\times 1)}{\eta}$$

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Comparison of frequency constants for $r/s = 4/3$

Case	No.	Exact	10 d.o.f. 4 d.o.f.	20 d.o.f. 4 d.o.f.
		Frequency constant		
1) Cantilever beam	1	3.52	3.52	3.52
	2	22.03	22.51	22.07
	3	61.70	64.98	62.77
2) Simply-supported beam	1	9.87	9.87	9.87
	2	39.48	39.54	39.92
	3	88.82	89.63	88.87
3) Free-free beam	1	22.37		22.42
	2	61.67		61.91
	3	120.91		123.60

Here η is a column vector of generalized coordinates. Then the reduced eigenvalue problem is

$$\tilde{D}\eta = \tilde{\eta} \quad (5)$$

where

$$\underset{(r\times r)}{\tilde{D}} = [T^t \underset{(r\times r)}{K} T]^{-1}\cdot[T^t \underset{(r\times r)}{M} T] \quad (6)$$

This is a drastically reduced problem not requiring significant inversions if the condensation is effective by the criterion described above.

For free-free vibration, rigid body modes A_0 can be adjoined to T as

$$q = [A_0 \quad T]\begin{Bmatrix}\eta_0\\ \eta\end{Bmatrix} \quad (7)$$

where A_0 is a rectangular matrix of rigid body modes.[5]

Operation of the modified transformation matrix on K and M gives

$$\tilde{D} = [T^t\ K\ T]^{-1}\big[T^t\ M\ T - [T^t\ M\ A_0]\times [A_0{}^t\ M\ A_0]^{-1}[A_0{}^t\ M\ T]\big]$$

with the relation

$$\eta_0 = -[A_0{}^t\ M\ A_0]^{-1}[A_0{}^t\ M\ T]\{\eta\} \quad (8)$$

Thus, the eigenvalue problem is reduced from n to $r \ll n$.

Unlike "relaxation" methods, no assumption of zero loads at condensed freedoms is implied. The major computational effort is already contained within the decomposition necessary for static analysis, and therefore should be less than for other condensation schemes.

Examples

To demonstrate the potential of this approach a few simple computations were made using finite beam elements: 1) a cantilever beam—5, 10 elements, 2) a simply supported beam—5, 10 elements, and 3) a free-free beam—10 elements, with condensation to a 4×4 matrix. Table 1 shows the very good agreement compared with the exact results.

Conclusions

The method suggests improvement in economy with reliability in both eigenvalue and eigenvector results. The comparative closeness of the ratio $r\div s$ to unity confirms a combination of efficiency with physical and mathematical appropriateness. The arbitrariness of choice of condensed freedoms is eliminated, with only the number of freedoms to be retained to be decided.

In a highly complex structural system, selection of the deformation vectors can be supplemented by an automated linear independency criterion which has been utilized in the example quoted. It would seem practical to extend the basic idea to include a rational approach to the handling of substructured problems. Thus, boundary choices might be automated and the over-all problem optimized.

References

[1] Argyris, J. H., "Some Results on the Free-Free Oscillations of Aircraft Type Structures," *Revue Francaise de Mecanique*, No. 15, 1965, pp. 59–73.

[2] Guyan, R. J., "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, Vol. 3, No. 2, Feb. 1965, pp. 380.

[3] Ojalvo, I. V. and Newman, M., "Vibration Modes of Large Structures by an Automatic Matrix-Reduction Method," *AIAA Journal*, Vol. 8, No. 7, July 1970, pp. 1234–1239.

[4] Lanczos, C., "An Iterative Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators," *Journal of Research*, Vol. 45, National Bureau of Standards, 1950, pp. 255–282.

[5] Argyris, J. H. and Kelsey, S., *Modern Fuselage Analysis and the Elastic Aircraft*, Butterworth, London, 1963, pp. 93–96.

Wind-Tunnel Magnus Testing of a Canted Fin or Self-Rotating Configuration

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Nomenclature

d = body diameter
m_p = yaw couple due to opposing forces $N\sin\varepsilon$ and N_p
p = spin in rad/sec
u = air velocity
C_n = measured yawing moment coefficient
C_N = normal force coefficient on the configuration $(N/\frac{1}{2}\rho u^2 S)$
C_{N_p} = Magnus force coefficient = $[N_p/\frac{1}{2}\rho u^2 S(pd/u)]$
C_Y = measured side force coefficient
N = normal force
N_p = Magnus force
N_s = normal force due to an angle of yaw
S = cross-sectional body area = $\pi d^2/4$
α = indicated angle of attack
β = angle of yaw at zero indicated angle of attack
ε = angle of roll of true angle-of-attack plane
ρ = air density
l = distance between $N\sin\varepsilon$ and N_p
l_F = Magnus force c.p. to c.g. distance
l_N = normal force c.p. to c.g. distance

Introduction

RECENT studies have shown that Magnus wind-tunne measurements on canted fin or self-rotating configuration: contain a normal force interaction term due to the inclinatioı of the pitch plane with respect to the balance measurin; directions. This interaction term is not the standard balanc interaction problem, but insteady requires that the amount o pitch plane inclination be calculated from zero spin pitch anı Magnus data. It is also possible in the case discussed in thi paper to make the correction after assuming the Magnus forc center of pressure location. The interaction term can be suffi

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